One-Dimensional, Steady-State Conduction with Thermal Energy Generation

Chapter Three

Section 3.5, Appendix C

Implications of Energy Generation

- Involves a local (volumetric) source of thermal energy due to conversion from another form of energy in a conducting medium.

- The source may be uniformly distributed, as in the conversion from electrical to thermal energy (Ohmic heating):

$$\dot{q} = \frac{\dot{E}_g}{\forall} = \frac{I^2 R_e}{\forall} \qquad (3.43)$$

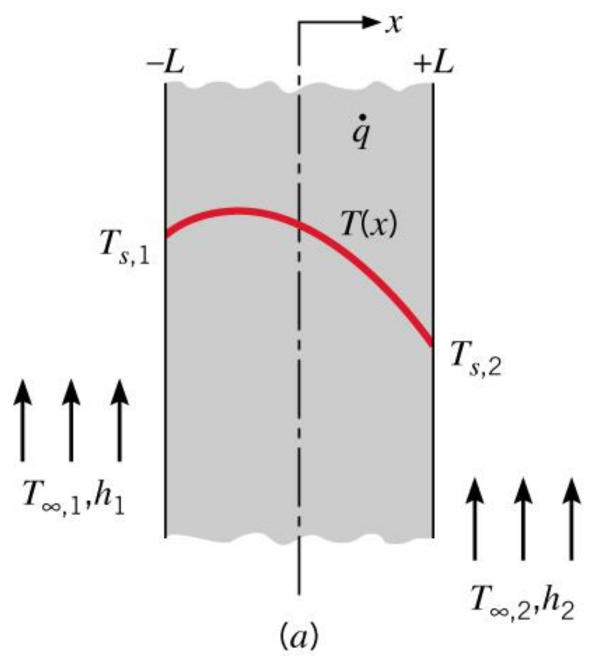
or it may be non-uniformly distributed, as in the absorption of radiation passing through a semi-transparent medium. For a plane wall,

$$\dot{q} \propto e^{-\alpha x}$$

- Generation affects the temperature distribution in the medium and causes the heat rate to vary with location, thereby precluding inclusion of the medium in a thermal circuit.

The Plane Wall

- Consider one-dimensional, steady-state conduction in a plane wall of constant k, uniform generation, and asymmetric surface conditions:



- Heat Equation:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+\dot{q}=0 \rightarrow \frac{d^2T}{dx^2}+\frac{\dot{q}}{k}=0 \qquad (3.44)$$

Is the heat flux q'' independent of x?

- General Solution:

$$T(x)=-(\dot{q}/2k)x^2+C_1x+C_2 \qquad (3.45)$$

What is the form of the temperature distribution for

$\dot{q}=0$? $\quad \dot{q}>0$? $\quad \dot{q}<0$?

How does the temperature distribution change with increasing $\dot{q}$?

Symmetric Surface Conditions or One Surface Insulated:

- What is the temperature gradient at the centerline or the insulated surface?
- Why does the magnitude of the temperature gradient increase with increasing x?
- Temperature Distribution:

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s \qquad (3.47)$$

- How do we determine T_s?

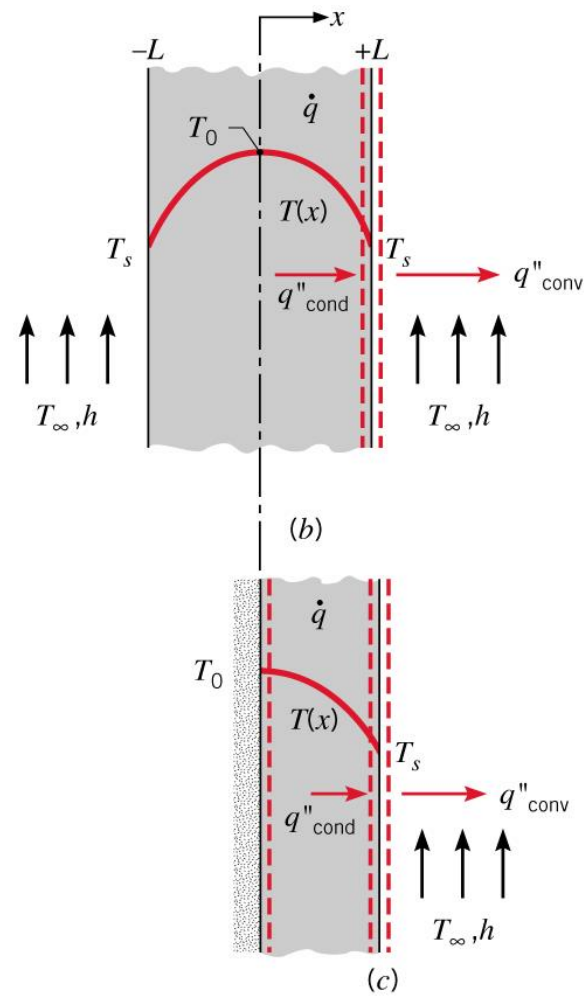
Overall energy balance on the wall →

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$-hA_s\left(T_s - T_\infty\right) + \dot{q}\,A_sL = 0$$

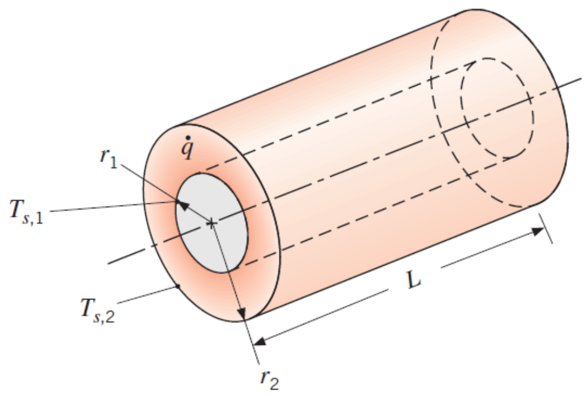
$$T_s = T_\infty + \frac{\dot{q}L}{h} \qquad (3.51)$$

- How do we determine the heat rate at $x = L$?

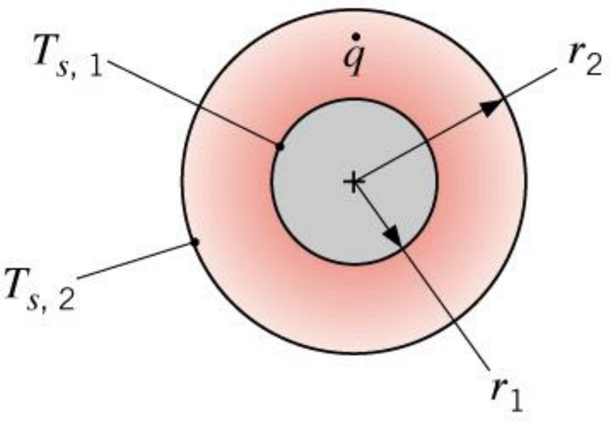


Radial Systems

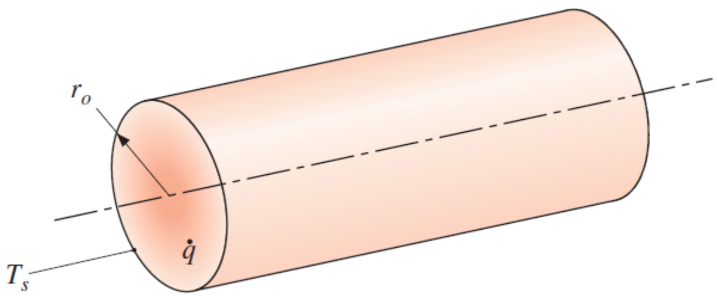
Cylindrical (Tube) Wall



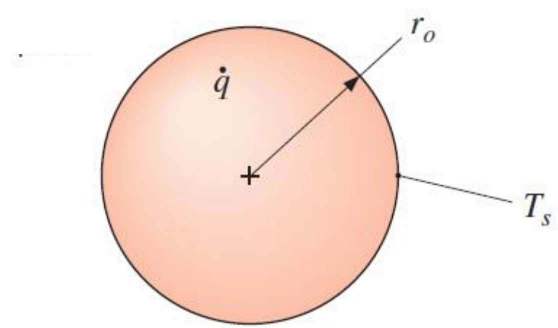
Spherical Wall (Shell)



Solid Cylinder (Circular Rod)



Solid Sphere



- Heat Equations:

Cylindrical

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)+\dot{q}=0$$

Spherical

$$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)+\dot{q}=0$$

- Solution for Uniform Generation in a Solid Sphere of Constant k with Convection Cooling:

Temperature Distribution

$$kr^2 \frac{dT}{dr} = -\frac{\dot{q} r^3}{3} + C_1$$

$$T = -\frac{\dot{q} r^2}{6k} - \frac{C_1}{r} + C_2$$

$$\frac{dT}{dr}\Big|_{r=0} = 0 \rightarrow C_1 = 0$$

$$T(r_o) = T_s \rightarrow C_2 = T_s + \frac{\dot{q} r_o^2}{6k}$$

$$T(r) = \frac{\dot{q} r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$$

Surface Temperature

Overall energy balance:

$$-\dot{E}_{\text{out}} + \dot{E}_g = 0 \rightarrow T_s = T_\infty + \frac{\dot{q} r_o}{3h}$$

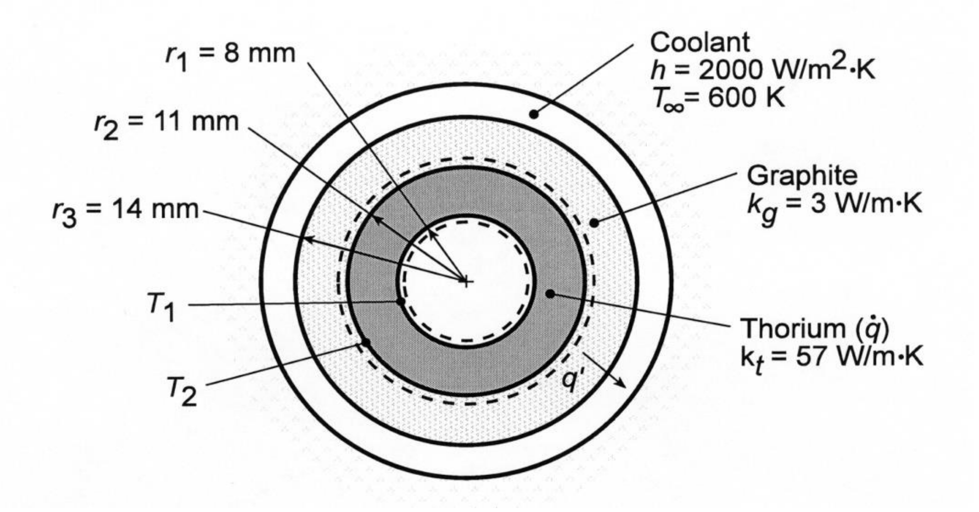
Or from a surface energy balance:

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0 \rightarrow q_{\text{cond}}(r_o) = q_{\text{conv}} \rightarrow T_s = T_\infty + \frac{\dot{q} r_o}{3h}$$

- A summary of temperature distributions is provided in Appendix C for plane, cylindrical and spherical walls, as well as for solid cylinders and spheres. Note how boundary conditions are specified and how they are used to obtain surface temperatures.

Problem 3.82 Thermal conditions in a gas-cooled nuclear reactor with a tubular thorium fuel rod and a concentric graphite sheath: (a) Assessment of thermal integrity for a generation rate of $\dot{q} = 10^8\,\mathrm{W/m^3}$. (b) Evaluation of temperature distributions in the thorium and graphite for generation rates in the range $10^8 \le \dot{q} \le 5\times10^8\ \mathrm{W/m^3}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

PROPERTIES: Table A.1, Thorium: $T_{mp} = 2023$ K; Table A.2, Graphite: $T_{mp} = 2273$ K.

ANALYSYS: (a) The outer surface temperature of the fuel, T_2 , may be determined from the rate equation

$$q' = \frac{T_2 - T_\infty}{R'_{\text{tot}}}$$

where $R'_{\text{tot}} = \frac{1\text{n}(r_3 / r_2)}{2\pi k_g} + \frac{1}{2\pi r_3 h} = 0.0185 \text{ m} \cdot \text{K/W}$

The heat rate may be determined by applying an energy balance to a control surface about the fuel element,

$$\dot{E}_{\text{out}} = \dot{E}_g$$

or, per unit length,

$$\dot{E}'_{\text{out}} = \dot{E}'_g$$

Since the interior surface of the thorium is essentially adiabatic, it follows that

$$q' = \dot{q}\pi\left(r_2^2 - r_1^2\right) = 17{,}907 \text{ W/m}$$

Hence,

$$T_2 = q'R'_{\text{tot}} + T_\infty = 17{,}907 \text{ W/m}\left(0.0185 \text{ m} \cdot \text{K/W}\right) + 600 \text{ K} = 931 \text{ K}$$

With zero heat flux at the inner surface of the fuel element, Eq. C.14 yields

$$T_1 = T_2 + \frac{\dot{q}r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) - \frac{\dot{q}r_1^2}{2k_t}1\text{n}\left(\frac{r_2}{r_1}\right) = 931 \text{ K} + 25 \text{ K} - 18 \text{ K} = 938 \text{ K}$$ <

Since T_1 and T_2 are well below the melting points of thorium and graphite, the prescribed operating condition is acceptable.

(b) The solution for the temperature distribution in a cylindrical wall with generation is

$$T_t(r) = T_2 + \frac{\dot{q} r_2^2}{4k_t}\left(1 - \frac{r^2}{r_2^2}\right) - \left[\frac{\dot{q} r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_2 - T_1)\right]\frac{1\text{n}(r_2/r)}{1\text{n}(r_2/r_1)} \quad \text{(C.2)}$$

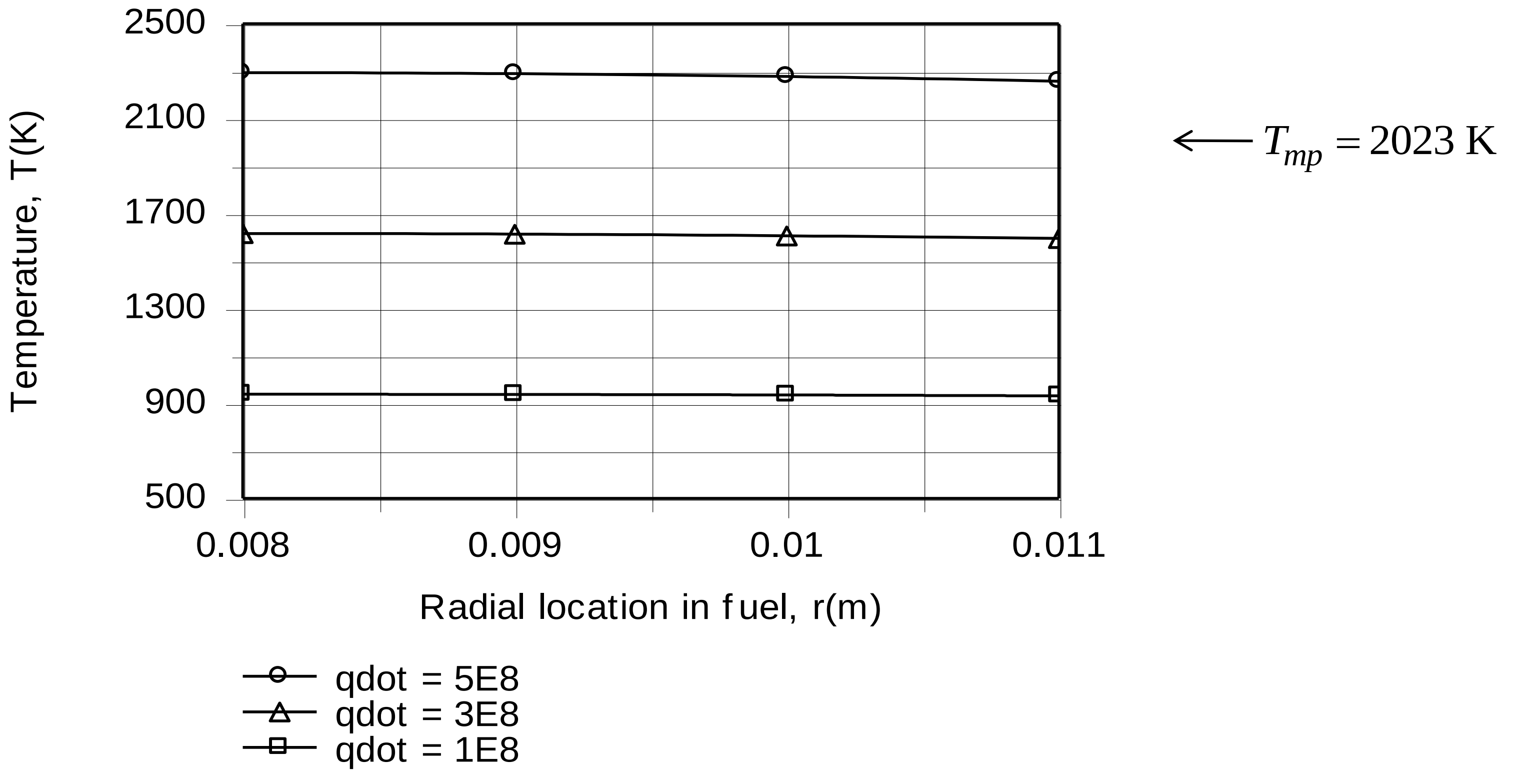
Boundary conditions at r_1 and r_2 are used to determine T_1 and T_2.

$$r = r_1: \qquad q_1'' = 0 = \frac{\dot{q} r_1}{2} - \frac{k\left[\frac{\dot{q} r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_2 - T_1)\right]}{r_1 1\text{n}(r_2 / r_1)} \quad \text{(C.14)}$$

$$r = r_2: \qquad U_2(T_2 - T_\infty) = \frac{\dot{q} r_2}{2} - \frac{k\left[\frac{\dot{q} r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_2 - T_1)\right]}{r_2 1\text{n}(r_2 / r_1)} \quad \text{(C.17)}$$

$$U_2 = \left(A_2' R_{\text{tot}}'\right)^{-1} = \left(2\pi r_2 R_{\text{tot}}'\right)^{-1} \quad \text{(3.37)}$$

The following results are obtained for temperature distributions in the thorium.



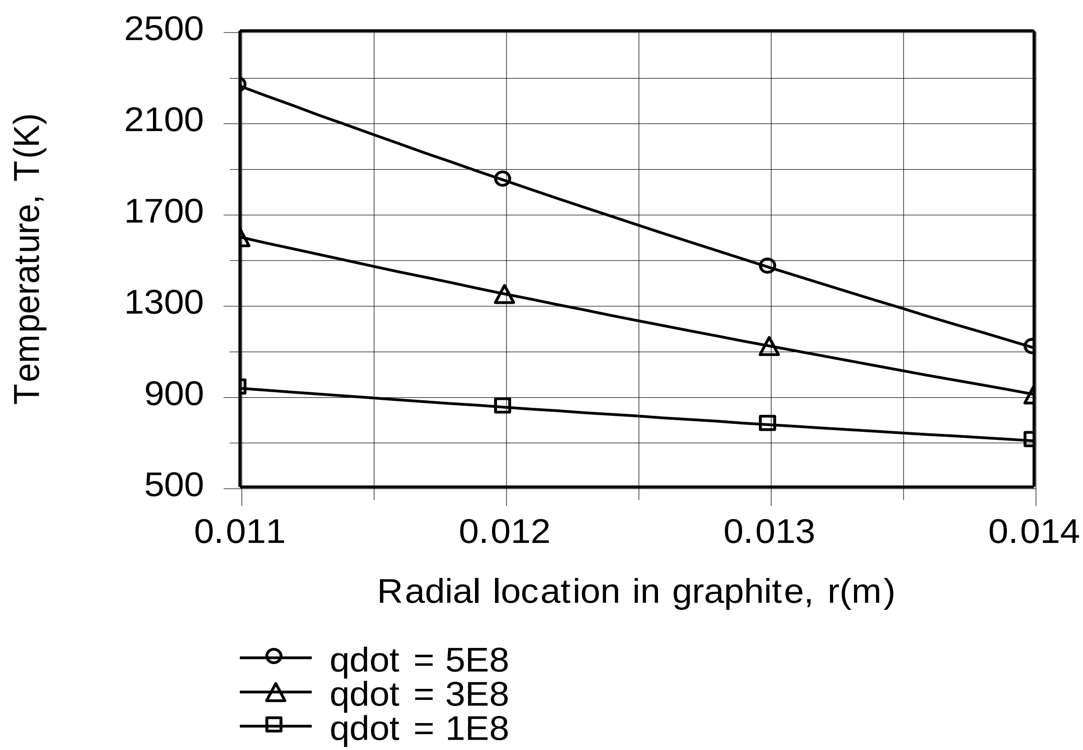
Operation at $\dot{q} = 5\times10^8\,\text{W/m}^3$ is clearly unacceptable since the melting point of thorium would be exceeded. To prevent softening of the material, which would occur below the melting point, the reactor should not be operated much above $\dot{q} = 3\times10^8\,\text{W/m}^3$. The small radial temperature gradients are attributable to the large value of k_t.

Using the value of T_2 from the foregoing solution and computing T_3 from the surface condition,

$$q' = \frac{2\pi k_g \left(T_2 - T_3\right)}{\ln\left(r_3 / r_2\right)}$$

the temperature distribution in the graphite is

$$T_g(r) = \frac{T_2 - T_3}{\ln\left(r_2 / r_3\right)} \ln\left(\frac{r}{r_3}\right) + T_3 \qquad (3.31)$$



$\longleftarrow T_{mp} = 2273\text{ K}$

Operation at $\dot{q} = 5 \times 10^8\ \mathrm{W/m^3}$ is problematic for the graphite. Larger temperature gradients are due to the small value of k_g.

COMMENTS: (i) What effect would a contact resistance at the thorium/graphite interface have on temperatures in the fuel element and on the maximum allowable value of $\dot{q}$? (ii) Referring to the schematic, where might radiation effects be significant? What would be the influence of such effect on temperatures in the fuel element and the maximum allowable value of $\dot{q}$?